An Inverse Problem Approach for Automatically Adjusting the Parameters for Rendering Clouds Using Photographs (supplemental document)

This document describes the details of the computation of the multiple scattering component.

Let us assume that a viewing ray through a pixel intersects with the cloud volume at $\mathbf{x}$ and $\mathbf{y}$, and the direction from $\mathbf{x}$ to the viewpoint is $\vec{\omega}$. We now formulate the intensity of light through point $\mathbf{x}$ in direction $\vec{\omega}$. We only consider the intensity due to scattering. Then, $L(\mathbf{x}, \vec{\omega})$ is expressed by:

$$L(\mathbf{x}, \vec{\omega}) = \int_{\mathbf{x}}^{\mathbf{y}} \beta \sigma_t \rho(\mathbf{x}') \tau(\mathbf{x}, \mathbf{x}') L_{sca}(\mathbf{x}', \vec{\omega}) d\mathbf{x}', \quad (1)$$

where β and σ_t are the albedo and the extinction cross section of the clouds particles, respectively. $\rho(\mathbf{x}')$ is the number density of cloud particles at $\mathbf{x}'$ and $\tau(\mathbf{x}, \mathbf{x}')$ is the attenuation of light between $\mathbf{x}$ and $\mathbf{x}'$ given by:

$$\tau(\mathbf{x}, \mathbf{x}') = \exp(-\int_{\mathbf{x}}^{\mathbf{x}'} \sigma_t \rho(\mathbf{x}'') d\mathbf{x}''). \quad (2)$$

$L_{sca}(\mathbf{x}', \vec{\omega})$ is expressed by:

$$L_{sca}(\mathbf{x}', \vec{\omega}) = \int_{\Omega} p(\vec{\omega} \cdot \vec{\omega}') L(\mathbf{x}', \vec{\omega}') d\vec{\omega}', \quad (3)$$

where Ω represents a set of directions on a unit sphere and p is the phase function.

We use a path tracing method for computing $L(\mathbf{x}, \vec{\omega})$. That is, $L(\mathbf{x}, \vec{\omega})$ is estimated as the average of a random variable $\bar{L}(\mathbf{x}, \vec{\omega})$, that is,

$$L(\mathbf{x}, \vec{\omega}) = E[\bar{L}(\mathbf{x}, \vec{\omega})]. \quad (4)$$

The average of $\bar{L}$ is evaluated by constructing many light paths. A light path is constructed in the following way. First, the location of a scattering event $\mathbf{x}'$ is determined by using a random number obeying the following probabilistic density function [Yue et al. 2010]:

$$P_{fp}(\mathbf{x}') = \sigma_t \rho(\mathbf{x}') \tau(\mathbf{x}, \mathbf{x}'). \quad (5)$$

If $\mathbf{x}'$ is outside the cloud volume, the path is discarded. Otherwise, the random variable $\bar{L}$ is evaluated by:

$$\begin{aligned} \bar{L}(\mathbf{x}, \vec{\omega}) &= \beta \sigma_t \rho(\mathbf{x}') \tau(\mathbf{x}', \mathbf{x}) L_{sca}(\mathbf{x}', \vec{\omega}) / P_{fp}(\mathbf{x}, \mathbf{x}') \\ &= \beta L_{sca}(\mathbf{x}', \vec{\omega}) \end{aligned} \quad (6)$$

L_{sca} is estimated in a similar way as the average of a random variable $\bar{L}_{sca}$. That is,

$$L_{sca}(\mathbf{x}', \vec{\omega}) = E[\bar{L}_{sca}(\mathbf{x}', \vec{\omega})]. \quad (7)$$

To evaluate the average of $\bar{L}_{sca}$, the scattering direction $\vec{\omega}'$ is determined using a random number obeying the probabilistic density function P_{dir}. Since the phase function is represented by the Henyey-Greenstein function, we can realize the importance sampling by using the function for P_{dir}. That is,

$$P_{dir}(\vec{\omega}') = p(\vec{\omega} \cdot \vec{\omega}'). \quad (8)$$

$\bar{L}_{sca}$ is then computed by:

$$\bar{L}_{sca}(\mathbf{x}', \vec{\omega}) = \frac{p(\vec{\omega} \cdot \vec{\omega}')}{P_{dir}(\vec{\omega}')} L(\mathbf{x}', \vec{\omega}') = L(\mathbf{x}', \vec{\omega}'). \quad (9)$$

$L(\mathbf{x}', \vec{\omega}')$ is evaluated recursively in the same way .

Our method computes the multiple scattering component based on the above formulation. However, in order to compute the multiple scattering within a practical computation time, the contribution of the sunlight directly reaching each scattering location is additionally taken into account. We assume that the sun is a point light source located at infinite distance. To evaluate the contribution of the direct sunlight, a point between each scattering location and the sun is randomly chosen using Eq. 5. If the point is outside the cloud volume, the contribution of the direct sunlight is accumulated.

The contribution of a single light path, Δ_l, is then computed by the following algorithm consisting of nine steps.

1. $k = 0$, $\Delta_l = 0$, $\Delta_{tmp} = 1$.
2. Determine the location of a scattering event, $\mathbf{x}'$, using Eq. 5.
3. If $\mathbf{x}'$ is outside the cloud volume and $k = 0$, then accumulate the contribution of the direct sunlight by:

   $$\Delta_l = L_{sun} \times p(\theta_{sun}),$$

   where L_{sun} is the intensity of the sun and θ_{sun} is the phase angle for the sun direction.
4. If $\mathbf{x}'$ is outside the cloud volume and $k > 0$, then terminate the path construction. Otherwise, proceed to the following steps.
5. $\Delta_{tmp} = \beta \times \Delta_{tmp}$
6. Generate point $\mathbf{x}''$ randomly between $\mathbf{x}'$ and the sun using the probabilistic density function in Eq. 5.
7. If $\mathbf{x}''$ is outside the cloud volume, accumulate the contribution of the direct sunlight by:

   $$\Delta_l = \Delta_l + \Delta_{tmp} \times L_{sun} \times p(\theta_{sun}),$$

8. Determine the scattering direction using Eq. 8.
9. If k is smaller than the user-specified number n_m, increment k by one and go to Step 3. Otherwise, terminate the path construction.

By computing the average of the contributions of many light paths using the above algorithm, the intensity of the pixel is obtained.

In our precomputation, β and L_{sun} are assumed to be unity and the contribution for each scattering order k is separately stored.

References

YUE, Y., IWASAKI, K., CHEN, B.-Y., DOBASHI, Y., AND NISHITA, T. 2010. Unbiased, adaptive stochastic sampling for rendering inhomogeneous participating media. *ACM Transactions on Graphics 29*, 6, Article 177.